Estimation of Process Parameters to Determine the Optimum Diagnosis Interval for Control of Defective Items

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The online quality monitoring procedure for attributes proposed by Taguchi has been critically studied and extended by a few researchers. Determination of the optimum diagnosis interval requires estimation of some parameters related to the process failure mechanism. Improper estimates of these parameters may lead to an incorrect choice of the diagnosis interval and thus huge economic penalties. We propose a Bayesian approach to estimate the process parameters under two different process models, commonly called as the case II and case III models in the literature. We discuss a systematic way to use available engineering knowledge in eliciting the prior for the parameters, and demonstrate the performance of the proposed method using extensive simulation and a case study from a hot rolling mill.

KEY WORDS: Economic model; Online monitoring; Quality; Statistical process control.

1. INTRODUCTION

Determination of the most economic sampling interval for control of defective items is a highly relevant problem to manufacturing processes that produce a continuous stream of products at high speed. On one hand, frequent inspection requires more cost, time, and manpower, whereas on the other hand, reduced frequency of inspection may lead to the risk of rejection of a large number of items. The problem of developing economically based online control methods for attributes has been addressed in detail by Taguchi (1981, 1984, 1985), Taguchi, Elsayed, and Hsiang (1989), and Nayebpour and Woodall (1993). The basic idea is to inspect a single item after every m units of production, with a process adjustment made as soon as a defective item is found. The value of m is to be determined to minimize the long-term expected cost per item under the assumption of fixed costs for each inspection, each defective item, and each process adjustment.

Primarily two typical patterns of arrival of special causes were considered. In the first case (referred to as case I in the literature), the process shifts from producing no defective items to producing all defective items. A detailed case study on a hot rolling process pertaining to an extension of case I has been given by Dasgupta (2003). In the second case (case II), the process shifts from producing no defective items to $100\pi\%$ defective items, where $0 < \pi < 1$. Note that case I is a special case of case II with $\pi = 1$. Nandi and Sreehari (1997) considered a combination of these two cases, designated case III, in which there are two types of assignable causes, termed minor and major, which appear according to a geometric pattern with parameters p_1 and p_2. The process shifts from a no-defective producing state to $100\pi\%$ or 100% defective producing state according as the minor or the major cause occurs.

Among other approaches, Adams and Woodall (1989) and Srivastava and Wu (1991, 1994, 1995, 1996) used random-walk models to analyze Taguchi's procedure. The problem of determination of optimal inspection intervals by minimizing the expected cost per unit of time over an infinite time span also has been studied in reliability literature (Barlow and Proschan 1965, chap. 4).

Whereas Taguchi did not explicitly assume a specific process failure mechanism (PFM), Nayebpour and Woodall (1993) considered a geometric PFM with a parameter p. This means that the duration of the process in the in-control state, measured by the number of items produced before a process shift occurs, is a geometric random variable with parameter p. The expected loss per item, $E(L)$, is a function of p in case I and (p, π) in case II. Thus the task of obtaining the optimal sampling interval comprises of the following two stages:

a. Estimate the parameters associated with the PFM from historical data.
b. Plug in these estimates into the expression for $E(L)$ and minimize it with respect to m.

Therefore, the solution to the optimization problem is strongly dependent on the estimate of the process parameters p and π. Biased estimation may lead to improper choice of the inspection interval and, consequently, huge economic penalties.

For both case II and case III, some prior information is available about one or more of the process parameters. This information may or may not be very accurate, but nonetheless it can be used to develop a Bayesian procedure for estimating the parameters. The Bayesian approach is natural in situations in which some experience-based prior knowledge exists. Harnessing such knowledge or information is likely to increase efficiency of the estimation. In this article we propose a Bayesian approach for estimating process parameters in cases II and III.

In Section 2 we review the underlying statistical model and optimization procedure associated with case II and introduce the notations used in the rest of the article. We devote Section 3

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to a brief discussion of the existing estimation procedures and the problems associated with them. In Section 4 we develop the Bayesian estimation procedure for case II and evaluate its performance using simulated examples. We also illustrate the approach with a case study. We propose and illustrate an estimation procedure for case III in Section 5 and present some concluding remarks in Section 6.

2. THE STATISTICAL MODEL AND OPTIMIZATION PROBLEM FOR CASE II

It is important to note that, as in the method of Taguchi (1981, 1984, 1985), the situation is different than typical control charting. Process monitoring is done not with control limits, but rather through individual product inspection. We also assume that the inspection error is negligible. Later, in Section 4.3, we briefly discuss how the estimation procedure may be affected by presence of type I and type II errors.

We assume that a cycle starts with the beginning of production or after an adjustment and ends with removal of the assignable cause. Let m denote the sampling interval. The following four components of cost are considered:

- C_d, the cost of producing one unit of defective product
- C_I, the cost of sampling and inspecting one unit of product
- C_a, the process adjustment cost expressed as $C_a = C_1 t + C_2$, where C_1 is the cost of stopping the process for one unit of time, t is the expected time for the adjustment, and C_2 is the direct recovery cost (See Nayebpour and Woodall 1993 for more details.)
- C_D, the cost of a defective item if it is sent to later stages of production or to the customer. Note that measurement of this cost component may not be straightforward. A short discussion of this issue is included in the concluding section.

We now introduce some notations that we use throughout the rest of the article. For the ith production cycle, $i = 1, 2, \ldots,$ we use the following:

- U_i, the number of products manufactured until the appearance of the first defect
- $X_i = \lfloor U_i/m \rfloor + 1$, the number of inspections from the beginning of the cycle to the first one immediately after the appearance of the first defect (Note that $\lfloor X \rfloor$ denotes the greatest integer contained in X [floor function].)
- Y_i, the number of additional inspections needed to detect the assignable cause after X_i
- l, the number of units produced from the time at which a defective item is sampled until the time that the production process is stopped for adjustment
- $S_i = X_i + Y_i + \lfloor l/m \rfloor$, the number of products inspected in the cycle
- $T_i = m(X_i + Y_i) + l$, the total length of a cycle or, in other words, the number of products manufactured in a cycle
- C_i, the total cost incurred in the cycle.

We assume that U_i and Y_i $(i = 1, 2, \ldots)$ are geometric random variables with parameters p and π so that

$$P(U_i = u) = pq^{u-1}, \qquad u = 1, 2, \ldots, \text{where } q = 1 - p,$$

and

$$P(Y_i = y) = \pi(1-\pi)^y, \qquad y = 0, 1, 2, \ldots.$$

It readily follows that the probability mass function (PMF) of X_i is given by

$$\begin{aligned} P(X_i = x) &= P\big((x-1)m < U_i \le mx\big) \\ &= q^{(x-1)m}(1 - q^m), \qquad x = 1, 2, \ldots. \end{aligned} \tag{1}$$

Let us consider a simple example (Fig. 1) to better characterize these variables. Suppose that the current sampling interval m is 10. The 17th item is the first defective item, after which the process starts producing $100\pi\%$ defectives. The 20th item is nondefective; thus the second inspection is unable to detect the assignable cause. The defect appears in the 30th item and is detected. But the process can be stopped after 4 more items have been manufactured, that is, after only the 34th item; thus in this cycle, $U = 17$, $X = 2$, $Y = 1$, $l = 4$, $T = 34$, and $S = X + Y = 3$.

Note that cases I and III also can be explained with the foregoing process model. As noted earlier, case I is a special case of case II where $\pi = 1$ and thus $Y = 0$ and $S = X + \lfloor l/m \rfloor$. In case III we have two possibilities—either the minor assignable cause (after which the process starts producing $100\pi\%$ defectives) or the major assignable cause (after which the process starts producing 100% defectives) appears first. Thus in this case $U = \min(U_1, U_2)$, where $U_1 \sim Geometric(p_1)$ and $U_2 \sim Geometric(p_2)$; consequently, $U \sim Geometric(p)$, where $p = p_1 + p_2 - p_1 p_2$.

The sequence $(T_1, C_1), (T_2, C_2), \ldots$ represents a renewal reward process (Ross 1996). Thus, by the renewal reward theorem, the long-term expected loss per product, $E(L)$, converges to $\frac{E(C)}{E(T)}$, where $E(C_i) = E(C)$ and $E(T_i) = E(T)$ for $i \ge 1$. Under the geometric PFM with a given p, explicit expressions for $E(C)$ and $E(T)$ can be computed (Nayebpour and Woodall 1993), and $E(L)$ can be expressed as a convex function of m for given p

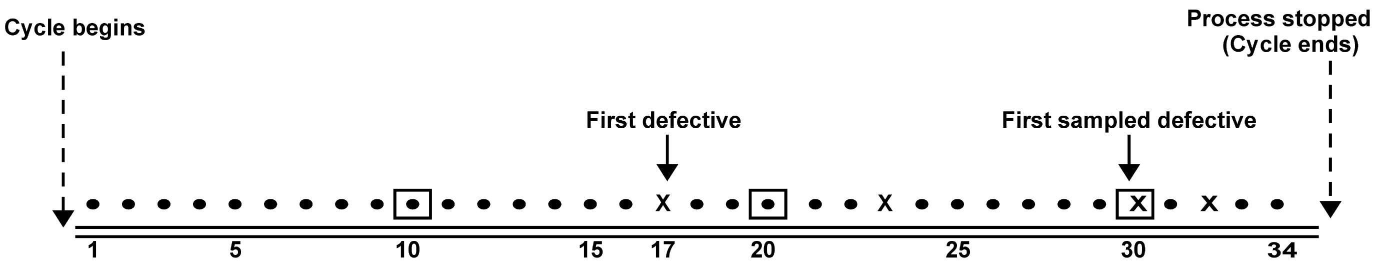

Figure 1. An illustration of case II (•, nondefective item; ×, defective item; □, inspected item).

and π; we denote this function by $L(m, p, \pi)$. Note that the expression for the loss function L depends on whether or not retrospective inspection is performed. To be specific, we can use the notation $L_1(m, p, \pi)$ in the case when retrospective inspection is performed and $L_2(m, p, \pi)$ when it is not. From Nayebpour and Woodall (1993), we have

$$L_1(m, p, \pi) = E_1(C)/E(T)$$

and

$$L_2(m, p, \pi) = E_2(C)/E(T),$$

where

$$\begin{aligned} E_1(C) = {} & \left(\left(m - \frac{q}{1-q} + \frac{mq^m}{1-q^m} \right) \pi^2 \right. \\ & \left. + m\pi(1-\pi) + l\pi \right) C_d \\ & + \left(\left(m - \frac{q}{1-q} + \frac{mq^m}{1-q^m} \right) \pi(1-\pi) \right. \\ & \left. + m(1-\pi)^2 \right) C_D \\ & + \left(\frac{m/(1-q^m) + m(1-\pi)/\pi}{m} + \left\lfloor \frac{l}{m} \right\rfloor \right) C_I + C_a, \\ E_2(C) = {} & \left(\left(m - \frac{q}{1-q} + \frac{mq^m}{1-q^m} \right) \pi + l\pi + m(1-\pi) \right) C_d \\ & + \left(\frac{m/(1-q^m) + m(1-\pi)/\pi}{m} + \left\lfloor \frac{l}{m} \right\rfloor \right) C_I + C_a, \end{aligned}$$

and

$$E(T) = \frac{m}{1-q^m} + \frac{m(1-\pi)}{\pi} + l.$$

Note that although both of the foregoing expressions include the adjustment cost component C_a, they are actually different in the two cases. The cost component C_a in the expression for $L_1(m, p, \pi)$ includes the cost of retrospective inspection, whereas that in $L_2(m, p, \pi)$ does not (see Nayebpour and Woodall 1993).

The moment estimator of p is given by

$$\hat{p} = 1 - \left(1 - \frac{m_c}{\overline{T} - (l + m_c(1-\pi)/\pi)} \right)^{1/m_c}, \qquad (2)$$

where $\overline{T}$ is the average of observed realizations $T_1, T_2, \ldots, T_N$, and m_c denotes the current sampling interval. Note that we need an estimate of π, say $\hat{\pi}$, to obtain $\hat{p}$; consequently, the optimum sampling interval is to obtained as

$$m^* = \arg\min L(m, \hat{p}, \hat{\pi}). \qquad (3)$$

Note that, to ensure that the estimate of p obtained from (2) is a real number satisfying $0 < \hat{p} < 1$, we must have

$$\pi > \frac{m_c}{\overline{T} - l}. \qquad (4)$$

We denote this lower bound of π by π_{bound}. This bound will be particularly useful because, from a practical standpoint, an engineer is more likely to have a preliminary knowledge of the range of π than of p, and, consequently, one we must use (2) to convert such knowledge to meaningful information on p.

3. EXISTING ESTIMATION METHODS

In case I the problem of estimation of p is straightforward, because we have an explicit expression for its maximum likelihood estimate (MLE) given by

$$\hat{p} = 1 - \left(1 - \frac{m_c}{\bar{T} - l} \right)^{1/m_c}. \qquad (5)$$

In case II the moment estimator of p given by (2) involves π, and biased estimation of π may lead to an erroneous estimate of p. Of course, estimation of π, becomes trivial if retrospective inspection is performed to trace back the starting point of the assignable cause, for example, if a defect is detected at the 40th product, and when traced back with 100% inspection (retrospective), the first defect is seen to occur in the 27th product. Then the 14 products (27–40) were produced after the occurrence of the special cause. This means that 14 is a realization of Y, which is a geometric random variable with parameter π. If more data on Y are generated in this way through retrospective inspections (with each production cycle generating one value of Y), then π can be estimated as $1/(1 + \bar{Y})$. Nayebpour and Woodall (1993) suggested that π should be estimated from such retrospective data.

But retrospective inspection is a costly affair; a company will do it only if it believes that a fairly high percentage of undetected defects likely will be passed on to the customer; that is, they perceive that the value of π is fairly high. Indeed, Nayebpour and Woodall (1993) recommended performing retrospective inspection if $C_I \le \pi C_D$, where C_I denotes the cost of sampling and inspecting one unit of product and C_D denotes the cost of a defective item if it is sent to later stages of production or to the customer. This may appear as a "chicken first or egg first" type of problem—retrospective inspection data are needed to estimate π and, on the other hand, an estimate of π is needed to determine whether or not to perform retrospective inspection.

To summarize, estimation of p and π is trivial if we perform retrospective inspection; however, the decision of whether or not to do retrospective inspection is not so trivial and ideally should depend on the value of π. Thus a direct (using the available data on cycle lengths) estimation method of the process parameters will have two advantages: It will help prevent economic penalties, as well as assist managers in making better decisions regarding whether or not to implement retrospective inspection.

The issue of optimal inspection policy for system failures, primarily from the standpoint of preventive maintenance, has been addressed in the reliability literature. The mathematical framework of case II described herein corresponds to the imperfect-inspection model discussed by Brown and Proschan (1983), Elsayed and Okumoto (1983), and Nakagawa (1988) for a single unit system. The probability of detecting failure in the imperfect-inspection model, β, is an analog of π, the probability of a defect occurring after the failure in case II.

A method of parameter estimation in an imperfect-inspection model was proposed by Srivastava and Wu (1993). They obtained a first-order binomial autoregressive model using an approximate likelihood filtering method under the assumption that $1 - \beta$ is small. Note that this is a good surrogate to case I, whereas in case II, $1 - \pi$ (or, equivalently, $1 - \beta$) is not small.

The method works as follows. Let x_k denote the number of failures detected out of n components by the kth inspection, $k = 1, 2, \ldots, K$ and let $\psi_k = p(1 - (1-\beta)^{\frac{x_{k-1}}{n}})$. Then, using the binomial autoregressive model given by

$$\sum_{k=1}^{K} x_k \ln(\psi_k) + (n - x_k)\ln(1 - \psi_k),$$

p and β are estimated by solving the following equations iteratively:

$$\frac{1}{K}\sum_{k=1}^{K}\frac{x_k/n}{\psi_k} = 1 \tag{6}$$

and

$$\frac{1}{K}\sum_{k=1}^{K}\frac{1 - x_k/n}{1-\psi_k} = 1. \tag{7}$$

In the present scenario, we have $n = 1$, with each x_k binary. Note that $x_k = 1$ if the kth inspection is the last inspection of a cycle and 0 otherwise. Assuming that $l = 0$, $\min_{i=1}^{N} s_i \geq 2$, and substituting π for β, after some algebra (see App. A), we obtain $\hat{p} = N/\sum_{i=1}^{N} s_i$ and $\hat{\pi} = 1$. Observe that the estimator of p is the same as its MLE in case I given by (5) when $m_c = 1$ and $l = 0$. This is expected, because case I is a special case of case II when $\pi = 1$. Thus, although the method works well for multiple-unit systems and large π, it may not be very appropriate for the generic case II problem, where $1 - \pi$ usually is not small, as seen in most real-life case examples in the quality control literature. The assumption $\min_{i=1}^{N} s_i \geq 2$ (which means that in the observed data, the first inspection of any new cycle does not detect a defect) is not very strong and holds well, for example, in the case study described in Section 4.4.

Moreover, it may be noted that conceptually, there is a difference between the inspection described in the reliability literature and that described in the quality control literature. Inspection for preventive maintenance involves inspecting system units or elements of the functional process, whereas the kind of inspection discussed here involves an inspection of product quality merely for symptoms of failure. Process monitoring in quality control usually comprises the following two stages: (a) detecting symptom of a failure through product inspection and (b) pinpointing the root cause by repeated checking of system components (see Dasgupta 2003 for a practical example). The optimal inspection policy described in the reliability literature can be successfully used in stage (b) whereas the current scenario deals with stage (a).

For case III, Nandi and Sreehari (1997) simply derived an expression for the expected loss and use it for optimizing the inspection interval, but did not discuss estimation of p_1, p_2, and π, which is a tricky problem. An analogy of case III with preventive maintenance in the reliability literature can be found by considering this case as a multiple shock model (Esary, Marshall, and Proschan 1973), in which both minor and major causes can occur independently. Estimation of process parameters in such a situation has not been discussed, however.

4. ESTIMATION OF p AND π IN CASE II

Suppose that we observe N production cycles and have the data on their lengths as $T_1, T_2, \ldots, T_N$, or, equivalently, on the number of products inspected in each cycle $s_1, s_2, \ldots, s_N$. The objective is to estimate π and p from this data set. To do this, we first consider the likelihood function of the parameters given by the following proposition (the proof of which is in App. A).

Proposition 1. The log-likelihood function of p and π is given by

$$\begin{aligned}\log L(p, \pi; s_1, s_2, \ldots, s_N) \\ &= N\log\pi + N\log(1 - q^m) - N\log|1 - \pi - q^m| \\ &\quad + \sum_{i=1}^{N}\log|(1-\pi)^{r_i} - q^{mr_i}|, \quad \text{where } r = s - \lfloor l/m \rfloor.\end{aligned}$$

Clearly, the log-likelihood does not yield a straightforward expression for the MLE. Thus we must use numerical methods to solve the optimization problem; however, owing to the complex nature of the nonlinear function, its direct optimization is not very easy.

It is highly likely that in most cases, process engineers will have some reasonable idea about π, which may not be good enough to check the condition $C_I \leq \pi C_D$ and determine whether retrospective inspection should be done, but may provide the analyst with a reasonable prior distribution for π. If similar information is available for p, then we may elicit a prior distribution for p as well. But, practically speaking, an engineer is more likely to answer the question of what percentage of defects the process produces when the process shifts with more conviction than the question of how long it takes for a process shift to occur.

The information obtained is usually in the form of an interval. In the next section we develop a formal methodology for translating such information into prior distributions for π and p. In the absence of prior information on p, we use (3) to convert the available knowledge about π to meaningful information on p.

4.1 Elicitation of Priors

Suppose, based on their past experience and/or a pilot study, that the process engineers are able to specify a reasonable range for π as $[\pi_L, \pi_U]$. Let $\overline{T}$ denote the mean cycle length as observed from the given data. Recalling from (4) that π must satisfy $\pi > \pi_{bound}$, we would assign a negligibly small mass of the prior distribution below π_{bound}. Therefore, we can elicit a $Beta(\alpha_\pi, \beta_\pi)$ prior for π where the hyperparameters can be obtained by solving

$$\frac{\Gamma(\alpha_\pi + \beta_\pi)}{\Gamma(\alpha_\pi)\Gamma(\beta_\pi)}\int_0^{\pi_{bound}} \pi^{\alpha_\pi - 1}(1-\pi)^{\beta_\pi - 1}\, d\pi = \epsilon \tag{8}$$

and

$$\frac{\Gamma(\alpha_\pi + \beta_\pi)}{\Gamma(\alpha_\pi)\Gamma(\beta_\pi)}\int_{\max(\pi_{bound}, \pi_L)}^{\pi_U} \pi^{\alpha_\pi - 1}(1-\pi)^{\beta_\pi - 1}\, d\pi = 1 - \gamma_\pi. \tag{9}$$

Clearly, (8) implies that there is a negligibly small probability, ϵ, that π will be less than π_{bound}, and (9) ensures that the probability of π lying beyond the stated interval equals a preassigned value γ_π. Note that $1 - \gamma_\pi$ can be interpreted as the *degree of belief* associated with the stated range. This degree of belief may be 100% if, for example, the engineer emphatically states that "I can guarantee that under no circumstances will π exceed 20%." In such a case, we can elicit a uniform prior for π over $[\max(\pi_{bound}, \pi_L), \pi_U]$.

Even when there is no available prior information on p, we can elicit a prior distribution for p based on the knowledge of π. Let p_L and p_U be the lower and upper limits of p obtained by substituting π_U and $\max(\pi_{bound}, \pi_L)$ in (2). Note that for $\pi > \pi_{bound}$, p and π are inversely related.

The hyperparameters α_p and β_p of a suitable beta prior distribution for p may be obtained by solving

$$\frac{\Gamma(\alpha_p+\beta_p)}{\Gamma(\alpha_p)\Gamma(\beta_p)}\int_0^{(p_L+p_U)/2} p^{\alpha_p-1}(1-p)^{\beta_p-1}\,dp = \frac{1}{2} \qquad (10)$$

and

$$\frac{\Gamma(\alpha_p+\beta_p)}{\Gamma(\alpha_p)\Gamma(\beta_p)}\int_{p_L}^{p_U} p^{\alpha_p-1}(1-p)^{\beta_p-1}\,dp = 1-\gamma_p. \qquad (11)$$

Note that (10) implies that the median of the distribution is taken at the midpoint of the interval $[p_L, p_U]$. Interpretation of (11) clearly is the same as that of (9).

4.2 Bayesian Estimation

Assuming that $p \sim Beta(\alpha_p, \beta_p)$ and $\pi \sim Beta(\alpha_\pi, \beta_\pi)$, it follows from Proposition 1 that for observed $s_1, s_2, \ldots, s_N$, the logarithm of the joint posterior density is given by

$$\log f(p,\pi; s_1,\ldots,s_n) \propto \begin{cases} (\alpha_p-1)\log p + (\beta_p-1)\log q + (N+\alpha_\pi-1)\log\pi \\ \quad + (\beta_\pi-1)\log(1-\pi) + N\log(1-q^m) \\ \quad - N\log|1-\pi-q^m| + \sum_{i=1}^{N}\log|(1-\pi)^{r_i} - q^{mr_i}| \\ \quad \text{for } 0\le p\le 1,\ 0\le\pi\le 1 \\ 0 \quad \text{otherwise.} \end{cases}$$

If we choose uniform priors $U(a_p, b_p)$ and $U(a_\pi, b_\pi)$ for p and π, then the logarithm of the joint posterior density is given by

$$\log f(p,\pi; s_1,\ldots,s_n) \propto \begin{cases} N\log\pi + N\log(1-q^m) - N\log|1-\pi-q^m| \\ \quad + \sum_{i=1}^{N}\log|(1-\pi)^{r_i} - q^{mr_i}| \\ \quad \text{for } a_p\le p\le b_p,\ a_\pi\le\pi\le b_\pi \\ 0 \quad \text{otherwise.} \end{cases}$$

Because $E(p,\pi|\mathbf{s})$, the Baye's estimate of (p,π), cannot be computed, we use Markov chain Monte Carlo (MCMC) methods (Gelman, Carlin, Stern, and Rubin 2004, chap. 11) to determine the parameter estimates. The target stationary distribution here is $f(p,\pi; s_1,\ldots,s_n)$. We cannot use Gibbs sampling, because it is not possible to derive the full conditional distributions of either of the parameters. We can use an appropriate Metropolis–Hastings algorithm to obtain the estimates, however.

We can choose $\pi^{(0)}$, the initial value of π, as the midpoint of the interval $[\pi_L, \pi_U]$ or $[\max(\pi_{bound}, \pi_L), \pi_U]$. The initial value of p can be obtained similarly, or by substituting $\pi = \pi^{(0)}$ in (4).

Alternatively, we may obtain maximum aposteriori (MAP) estimates by maximizing the posterior density (Gelman et al. 2004, chap. 12). However, the complexities involved in this optimization are likely to be at least equal to those involved with optimization of the original likelihood function. In fact, with uniform priors, the problem of maximization of the posterior likelihood becomes identical to direct maximization of the likelihood with tighter constraints.

4.3 A Simulated Example

Consider a process in which we have $p = .000339$, as in the case I example of Nayebpour and Woodall (1993). Let $m_c = 500$, $\pi = .10$, and $l = 0$. We simulate 200 production cycles from the foregoing process, thereby generating data of the form $s_1, s_2, \ldots, s_{200}$.

Regarding the available prior information, we consider the following two situations:

- The process engineer, based on his or her experience, states that "when the process goes out of control, it produces at most 15% defectives on an average. I have no idea about p."
- The process engineer states that "as per my experience, the appropriate range for π is $12 \pm 5\%$. p usually doesn't exceed .0005."

To obtain estimates of p and π in each of these two situations, we use three different priors: (a) uniform prior, a tight Beta prior with $\gamma_\pi = \gamma_p = .05$, and a flatter Beta prior with $\gamma_\pi = \gamma_p = .25$. To obtain the hyperparameters, we proceed as follows. In situation 1, we have $\pi_L = 0$ and $\pi_U = .15$. Noting that $\bar{s} = 15.275$, we obtain $\pi_{bound} \approx .07$. All of the methods require some starting values of p and π. As stated in Section 4.2, we can choose $(\pi_L + \pi_U)/2 = .075$ as the starting value $\pi^{(0)}$, because it is greater than π_{bound}. An appropriate range for π to set up the priors thus would be $[\max(\pi_L, \pi_{bound}), \pi_U]$, which in this case is $[.07, .15]$. Although we have no prior information on p, plugging the foregoing limits into (2), we obtain the interval $[.0002, .0009]$. The starting value $p^{(0)}$ can be taken as the midpoint of this interval, that is, .00055.

In situation 2, we have $\pi_L = .07$ and $\pi_U = .17$. Because $\bar{s} = 15.275$, we obtain $\pi_{bound} \approx .07$. The midpoint of the stated range .12 can be chosen as a starting point for π. The initial value of p can be chosen as .00025. To set up the priors, appropriate ranges for π and p will be $[.07, .17]$ and $[.0002, .0005]$.

The prior distributions with hyperparameters for the six cases (two situations, with three priors corresponding to each) are given in Table 1. Note that the hyperparameters for the Beta priors are derived from (8)–(11), taking $\epsilon = .001$.

For each of the six cases listed in Table 1, two different estimators of p and π were used, an MCMC-based estimator and the MAP estimator. To obtain the former, the mcmc library in R, which uses a Metropolis algorithm, was used to perform the MCMC simulations. Each MCMC run consisted of 10,000 iterations with a burn-in of 1,000. To obtain the posterior mode, the posterior likelihood was maximized using the optim function in R (R Development Team 2006).

Table 1. Prior distributions for p and π for the Bayesian methods

Prior		Degree of belief $1-\gamma$	Situation 1 Prior for p	Situation 1 Prior for π	Situation 2 Prior for p	Situation 2 Prior for π
Uniform			U[.0002, .0009]	U[.07, .15]	U[.0002, .0005]	U[.07, .17]
Beta	Tight	95%	*Beta*(9, 16355)	*Beta*(25, 196)	*Beta*(20, 56500)	*Beta*(20, 144)
	Flat	75%	*Beta*(3, 5000)	*Beta*(19, 125)	*Beta*(6.75, 18550)	*Beta*(11, 76)

The simulation was repeated 100 times; the results are summarized in Tables 2 and 3. Denoting the estimate of p from the ith simulation ($i = 1, 2, \ldots, 100$) by $\hat{p}_i$ and the true value by p_0, the tables report the mean, median, mode, and standard deviation of $\hat{p}_1, \hat{p}_2, \ldots, \hat{p}_{100}$, and percentage relative bias, given by

$$\text{relative bias} = \frac{(1/100)\sum_{i=1}^{100} \hat{p}_i - p_0}{p_0} \times 100\%.$$

From these results, we can make the following observations:

1. The MAP estimates were poor in both situations, irrespective of the prior distributions. Table 4 shows that the likelihood had multiple maxima even in the small exploration range $.00025 \le p \le .00060$ and $.07 \le \pi \le .14$. This led to multiple modes in the posterior distribution. Convergence of the optimization algorithm is seen to depend on the initial choices of the parameters.
2. The Bayes estimates obtained using MCMC performed better and were more robust to the varying levels of available preliminary information on the parameters.
3. For the MCMC-based estimates, the relative bias and variance of $\hat{p}$ corresponding to almost every method were generally seen lower in situation 2 compared with situation 1, which shows that, as expected, with better and more accurate prior information, more efficient estimates can be obtained with lower bias.

Effect of Current Sampling Interval (m_c). In the foregoing simulation example, the parameter m_c (current sampling interval) was taken as 500, because the optimal interval for this case (Nayebpour and Woodall 1993) was around 500. We now extend our simulation to cases in which m_c is different than 500 and study the robustness of the proposed estimation method against variations in the initial sampling interval. Note that a statistician usually will have no control over m_c, because it denotes the current practice adopted by the process engineers. Table 5 gives simulation results for $m_c = 50$, 100, 500, 1,000, and 5,000. Except for the last case (i.e., $m_c = 5{,}000$), the results were more or less satisfactory. Of course, 5,000 is an unrealistic value of m_c in this example. In general, we found that as m_c increased, the precision (inverse of the sampling error) of the estimate of p decreased, whereas that of π increased. An intuitive explanation can be provided for such behavior. If $m_c = 1$ (i.e., every item is sampled), then we would obtain a very precise estimate of p but a poor estimate of π, because the very first defect will be detected. The reverse would occur if m_c were very large. Note that Table 5 includes only the MCMC estimates, because the MAP estimates are not recommended based on the results in Tables 2 and 3.

Effect of Sample Size (N). Although from a practical standpoint, the issue of sample size is usually important in most estimation problems, it may not be very important here, because the estimation procedure does not require generation of fresh data, but needs only historical data. The quality department of any organization will have plenty of past inspection records that can be used for estimation.

Clearly, it is difficult to provide a theoretical solution to the problem of determination of optimal sample size in the present context. To obtain an idea of the optimal sample size in the

Table 2. Summary of simulation output in case II: Estimation of p

Available info	Method			mean($\hat{p}$) $\times 10^{-4}$	median($\hat{p}$) $\times 10^{-4}$	mode($\hat{p}$) $\times 10^{-4}$	relative bias($\hat{p}$)%	sd($\hat{p}$) $\times 10^{-5}$
$0 \le \pi \le .15$	Uniform		MCMC	3.64	3.35	3.02	7.39	9.1
No info on p			MAP	5.94	5.93	5.99	75.13	4.9
	Beta	Tight	MCMC	3.80	3.69	3.44	12.10	6.2
			MAP	2.08	2.00	2.00	−38.57	7.3
		Flat	MCMC	3.04	2.86	2.71	−10.28	6.7
			MAP	2.08	2.00	2.00	−38.53	7.6
$.07 \le \pi \le .17$	Uniform		MCMC	3.16	3.05	2.92	−6.91	4.3
$0 < p \le .0005$			MAP	4.89	5.00	5.00	44.25	5.7
	Beta	Tight	MCMC	3.19	3.14	3.06	−6.04	2.6
			MAP	4.97	5.00	5.00	46.72	2.8
		Flat	MCMC	3.20	3.10	2.86	−8.55	6.2
			MAP	4.99	5.00	5.00	47.49	1.9

Table 3. Summary of simulation output in case II: Estimation of π

Available info	Method			Estimate of π mean($\hat{\pi}$) $\times 10^{-2}$	median($\hat{\pi}$) $\times 10^{-2}$	mode($\hat{\pi}$) $\times 10^{-2}$	relative bias($\hat{\pi}$)%	sd($\hat{\pi}$) $\times 10^{-3}$
$0 \leq \pi \leq .15$ No info on p	Uniform		MCMC	10.12	10.21	11.04	1.25	11.2
			MAP	8.26	8.38	8.52	−17.41	5.4
	Beta	Tight	MCMC	9.83	9.90	10.11	−1.65	7.2
			MAP	7.12	7.00	7.00	−28.84	9.2
		Flat	MCMC	11.31	11.57	11.68	13.07	8.9
			MAP	7.10	7.00	7.00	−29.04	8.7
$.07 \leq \pi \leq .17$ $0 < p \leq .0005$	Uniform		MCMC	10.80	10.93	11.38	7.97	10.2
			MAP	16.72	17.00	16.99	67.20	16.5
	Beta	Tight	MCMC	10.70	10.72	10.76	6.98	6.9
			MAP	16.92	17.00	16.99	69.18	8.9
		Flat	MCMC	10.68	10.66	10.64	6.80	10.4
			MAP	16.96	17.00	17.00	69.60	6.3

current simulation example, we study the effect of sample size on the mean squared error (MSE) of the estimates. Recall that from 100 simulations, we had estimated the relative bias and standard error for the estimated parameters. Now we vary the sample size N (number of cycles) from 10 to 300 in steps of 10 and compute the MSE as the sum of squared absolute bias and variance. Figure 2 shows the plot of estimated $MSE(\hat{p})$ versus N, corresponding to situation 1 with a uniform prior. From the continuous curve fitted through these points, we conclude that a sample size of about 150 should be good enough, because the reduction in MSE after 150 is not very significant. On the other hand, $MSE(\hat{\pi})$ (not shown here) stabilizes much faster. The results are similar for situation 2 and the Beta priors.

Effect of Inspection Error. Although in the development of the Bayesian estimation methodology, we have assumed that the inspection error is negligible, this may not be the case in all situations. Borges, Ho, and Turnes (2001) developed an extension of the problem in the presence of diagnostic errors. As they showed, extension of the results of Nayebpour and Woodall (1993) requires the development of a new model. The optimal monitoring procedures are more sensitive to changes in type I error than to changes in type II error. But Borges et al. (2001) assumed that p and π are known. Here we show how the presence of type I and type II inspection errors affect the estimation of p and π and thus the optimal monitoring procedure.

We simulated the same process as described in Section 4.3 with type I error (denoted by a) ranging from .001 to .05 and type II error (denoted by b) ranging from .005 to .10. The detailed results are given in Tables A.1 and A.2 in Appendix B. For both p and π, the bias of the estimators was more significantly affected by type I error than by type II error. Therefore, in a situation in which a significant inspection error is expected, the estimation method should be modified. This is a topic for future research.

4.4 Case Study

Although quite a few case studies on the case II problem (Taguchi et al. 1989, p. 111) have been reported, none of these has reported the complete raw data on the cycle lengths. Owing to the absence of such published raw data that pertain exactly to the stated scenario, we consider the hot rolling case study reported by Dasgupta (2003). Certain defects do not appear in each hot-rolled product after the process shifts. For example, when a guide connecting two successive rolling stands becomes old or worn out, every product passing through it will not necessarily be defective; however, there is a moderately high probability, π, that it will. Therefore, a hot rolling process perfectly fits into the case II setup as well.

Table 4. Likelihood with multiple maxima

p	π .07	.08	.09	.10	.11	.12	.13	.14
.00025	−752.08	−743.38	−737.38	−733.39	−730.89	−729.49	−728.90	−728.91
.00030	−744.20	−736.81	−732.23	−729.74	−728.81	−729.04	−730.13	−731.83
.00035	−739.17	−732.93	−729.59	−728.42	−728.91	−730.66	−733.35	−736.71
.00040	−735.86	−730.62	−728.34	−728.32	−730.06	−733.15	−737.27	−742.17
.00045	−733.65	−729.25	−727.89	−728.87	−731.67	−735.92	−741.30	−747.55
.00050	−732.16	−728.49	−727.91	−729.74	−733.46	−738.70	−745.16	−752.58
.00055	−731.14	−728.10	−728.20	−730.76	−735.28	−741.37	−748.76	−757.19
.00060	−730.44	−727.95	−728.64	−731.84	−737.04	−743.88	−752.06	−761.35

Table 5. Summary of simulation output: Effect of m_c

				Estimate of p			Estimate of π		
m_c	Available information	Method		$mean(\hat{p})$ $\times 10^{-4}$	$relative$ $bias(\hat{\pi})\%$	$sd(\hat{p})$ $\times 10^{-5}$	$mean(\hat{\pi})$ $\times 10^{-2}$	$relative$ $bias(\hat{\pi})\%$	$sd(\hat{\pi})$ $\times 10^{-3}$
50	$0 \le \pi \le .15$ No information on p	Uniform		3.41	.68	2.50	10.51	5.09	13.99
		Beta	Tight	3.44	1.44	2.40	10.69	6.90	9.51
			Flat	3.35	−1.04	2.32	12.03	20.28	12.39
	$.07 \le \pi \le .17$ $0 < p \le .0005$	Uniform		3.38	−.16	2.44	11.01	10.14	17.61
		Beta	Tight	3.32	−2.19	2.25	12.52	25.17	17.46
			Flat	3.36	−.81	2.18	11.37	13.70	11.65
100	$0 \le \pi \le .15$ No information on p	Uniform		3.44	1.34	2.96	10.35	3.45	14.04
		Beta	Tight	3.45	1.91	2.72	10.47	4.66	9.77
			Flat	3.33	−1.79	2.69	11.47	14.71	12.91
	$.07 \le \pi \le .17$ $0 < p \le .0005$	Uniform		3.38	−.40	2.93	10.74	7.43	18.06
		Beta	Tight	3.26	−3.89	2.54	12.04	20.45	17.47
			Flat	3.34	−1.45	2.38	11.05	10.52	11.77
500	$0 \le \pi \le .15$ No information on p	Uniform		3.59	5.88	8.81	10.14	1.36	11.24
		Beta	Tight	3.76	10.91	5.90	9.83	−1.68	7.15
			Flat	3.04	−10.22	6.60	11.35	13.53	9.04
	$.07 \le \pi \le .17$ $0 < p \le .0005$	Uniform		3.16	−6.93	4.20	10.82	8.15	10.41
		Beta	Tight	2.68	−21.01	2.80	12.26	22.64	6.01
			Flat	3.18	−6.29	2.65	10.67	6.73	6.99
1000	$0 \le \pi \le .15$ No information on p	Uniform		3.87	14.01	11.02	10.04	.39	9.74
		Beta	Tight	4.00	17.94	7.74	9.95	−.50	7.08
			Flat	3.49	2.88	10.76	10.54	5.44	10.54
	$.07 \le \pi \le .17$ $0 < p \le .0005$	Uniform		3.39	−.05	5.69	10.29	2.91	9.41
		Beta	Tight	3.16	−6.85	6.89	10.76	7.65	10.92
			Flat	3.32	−2.12	3.73	10.30	3.01	7.05
5000	$0 \le \pi \le .15$ No information on p	Uniform		5.00	47.46	11.16	9.99	−.09	6.61
		Beta	Tight	4.98	46.78	6.46	10.06	.61	5.94
			Flat	4.89	44.36	16.05	10.22	2.20	6.56
	$.07 \le \pi \le .17$ $0 < p \le .0005$	Uniform		3.47	2.39	4.11	10.10	.97	6.94
		Beta	Tight	3.43	1.21	6.00	10.28	2.77	6.80
			Flat	3.46	2.05	2.47	10.22	2.17	6.41

Although this process is actually an extended version of case I, we can visualize it as a case II example with $\pi = p_v$ and $Y = V$. In the original problem, V is defined as the number of items produced starting from detection of the defect to identification of the specific process fault, and it follows a geometric distribution with parameter p_v. Although we have separate data on X and Y for each of the 100 production cycles, we assume that we observe only T, that is, the total cycle length. Table 6 gives the complete data. Note the following:

a. Because only T is assumed to be observed, we obtain $S = [\frac{T-l}{m_c}]$.

b. Because $[l/m_c] = 0$, we have $S = X + Y$.

c. l actually is a random variable, which is seen to vary between 3 and 5; however, we assume it to be a constant equal to 4.

We assume that when asked about π, the process engineer states that “it doesn’t exceed 12%.” Because we have no lower bound for π, we take $\pi_L = \pi_{bound} \approx .06$. Substituting π_L and π_U in 2, we get $p_L = .0095$ and $p_U = .046$.

Using the Bayesian algorithm with a uniform prior over the intervals [.06, .12] and [.0095, .046], we obtain the estimates of π and p as $\hat{\pi} = .0888$ and $\hat{p} = .0132$. Because no ret-

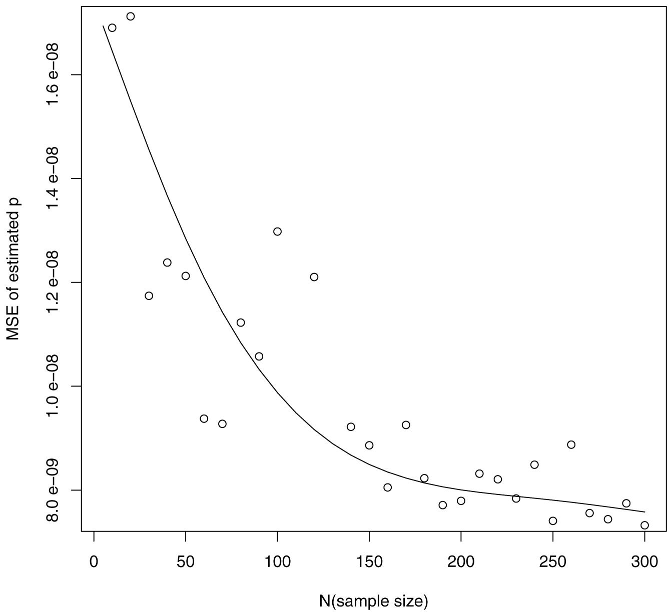

Figure 2. Sample size versus estimated MSE of $\hat{p}$.

rospective inspection is performed, we use the loss function $L_2(m, \hat{p}, \hat{\pi})$ for optimization. Substituting $C_I = 21$, $C_D = 138$, and $C_a = 100$, we obtain the optimum diagnosis interval as $m^* = 3$. The result remains the same if we take $\pi_L = .07$ instead of .06.

Note that if we use the data on X and Y separately, then we can easily obtain that $\hat{\pi} = .0857$, $\hat{p} = .0139$, and subsequently, $m^* = 3$.

5. ESTIMATION OF PARAMETERS IN CASE III

Case III, discussed by Nandi and Sreehari (1997), deals with a scenario in which there are two types of assignable causes, termed minor and major, and their appearances follow geometric patterns with parameters p_1 and p_2. The occurrence of a major assignable cause leads to a situation like case I, in which all subsequent items produced are defective. A minor assignable cause leads to a situation like case II, that is, the process starts producing $100\pi\%$ defective products after the occurrence of such a cause. Although Nandi and Sreehari (1997) derived expressions for the expected loss, they completely ignored the estimation of p_1, p_2, and π.

Once again, a real-life example can be given from a hot rolling mill consisting of a sequence of rolling stands. A problem in any of the rolling stands generally would lead to 100% defectives, whereas if a guide or a roller were worn out, this could result in a defective product with a certain probability.

Note that for this case, we can use exactly the same notation as in Section 2 if we define $p = p_1 + p_2 - p_1 p_2$, that is, $q = q_1 q_2$, where $q_i = 1 - p_i$ for $i = 1, 2$. We assume that as in case II, the data would be of the form $s_1, s_2, \ldots, s_N$, that is, the number of inspections conducted in each of N production cycles.

5.1 Bayesian Estimation of p_1, p_2, and π

To develop the Bayesian algorithm, we need the following result, the proof of which is in Appendix A.

Table 6. Data from the hot rolling case study

Cycle number	X	Y	T	S	Cycle number	X	Y	T	S
1	4	20	243	24	51	1	4	53	5
2	10	10	203	20	52	2	23	254	25
3	1	2	35	3	53	10	4	144	14
4	2	13	155	15	54	9	0	94	9
5	7	1	85	8	55	1	9	104	10
6	32	6	384	38	56	5	38	435	43
7	1	25	264	26	57	2	17	194	19
8	2	0	25	2	58	11	30	414	41
9	6	20	264	26	59	18	16	345	34
10	1	5	64	6	60	4	5	94	9
11	23	20	435	43	61	1	6	74	7
12	2	32	344	34	62	1	26	274	27
13	8	12	204	20	63	7	11	185	18
14	1	3	45	4	64	6	40	463	46
15	3	5	84	8	65	3	14	173	17
16	1	0	14	1	66	7	0	74	7
17	6	18	243	24	67	2	0	25	2
18	23	24	474	47	68	14	10	243	24
19	4	6	104	10	69	5	28	333	33
20	4	7	113	11	70	26	48	743	74
21	12	7	193	19	71	3	9	124	12
22	26	16	424	42	72	14	8	225	22
23	2	3	53	5	73	7	2	93	9
24	6	9	155	15	74	1	3	43	4
25	5	4	95	9	75	3	5	85	8
26	14	10	244	24	76	3	0	34	3
27	16	0	165	16	77	13	2	153	15
28	5	11	164	16	78	6	14	204	20
29	14	9	235	23	79	4	0	45	4
30	17	7	244	24	80	12	7	193	19
31	4	3	75	7	81	33	17	505	50
32	1	11	123	12	82	2	10	125	12
33	6	18	243	24	83	20	14	344	34
34	3	0	34	3	84	2	13	153	15
35	1	5	64	6	85	12	9	213	21
36	1	3	43	4	86	2	18	204	20
37	6	7	135	13	87	21	3	244	24
38	2	4	64	6	88	11	4	154	15
39	2	18	205	20	89	5	2	73	7
40	16	7	234	23	90	11	12	233	23
41	2	6	84	8	91	1	7	83	8
42	7	6	133	13	92	19	25	444	44
43	6	5	115	11	93	24	10	343	34
44	14	14	284	28	94	19	8	273	27
45	2	46	483	48	95	1	13	145	14
46	3	9	124	12	96	1	8	94	9
47	1	4	54	5	97	9	1	104	10
48	1	10	114	11	98	14	9	234	23
49	5	2	73	7	99	6	7	134	13
50	4	5	93	9	100	7	10	175	17

Proposition 2. The probability mass function of S is given by

$$P(S = s) = (1 - q^m)\Big[\alpha(1 - \pi)(\pi q_2^m + 1 - q_2^m)$$

$$\times \Delta\left(\frac{\{(1-\pi)q_2^m\}^{r-1} - q^{m(r-1)}}{(1-\pi)q_2^m - q^m}\right)$$

$$+ q^{m(r-1)}(1 - \Delta\alpha(1-\pi))\Big], \qquad s = 1, 2, \ldots, \infty,$$

where

$$r = s - \lfloor l/m \rfloor,$$
$$p = p_1 + p_2 - p_1 p_2,$$
$$\alpha = \frac{p_1 - p_1 p_2}{p},$$

and

$$\Delta = q_2 . \frac{1 - q_1}{1 - q_1^m} . \frac{q_2^m - q_1^m}{q_2 - q_1}.$$

We assume that $p_i \sim Beta(\alpha_i, \beta_i)$, $i = 1, 2$, and $\pi \sim Beta(\alpha_3, \beta_3)$. Then Proposition 2 leads to the following corollary.

Corollary 1. Let $g(p_1, p_2, \pi | s_1, \ldots, s_N)$ denote the joint posterior distribution of (p_1, p_2, π), where the priors are the Beta distributions defined earlier. Let r, p, and Δ be as defined in Proposition 3. Then

$$\begin{aligned} &\log g(p_1, p_2, \pi | s_1, \ldots, s_N) \\ &\quad \propto N \log(1 - q^m) \\ &\qquad + \sum_{i=1}^{N} \log\Big[\alpha(1-\pi)(\pi q_2^m + 1 - q_2^m) \\ &\qquad \times \Delta \frac{\{(1-\pi) q_2^m\}^{r_i - 1} - q^{m(r_i - 1)}}{(1-\pi) q_2^m - q^m} \\ &\qquad + q^{m(r_i - 1)}\{1 - \Delta\alpha(1-\pi)\}\Big] \\ &\qquad + \sum_{j=1}^{2} (\alpha_j - 1) \log p_j + (\alpha_3 - 1) \log \pi \\ &\qquad + \sum_{j=1}^{2} (\beta_j - 1) \log q_j + (\beta_3 - 1) \log(1 - \pi). \end{aligned}$$

Like case II, here we need to elicit prior distributions for all three parameters p_1, p_2, and π. Assuming that we have some lower and upper bounds for each of the three parameters, the hyperparameters α_i, β_i, $i = 1, 2, 3$, can be obtained in the same way as discussed in Section 4.1. If the engineers are more or less certain about the limits and are unable to say anything more about the prior distributions, then uniform priors could be a possible choice again.

Considering the complications involved in finding the posterior modes, we use MCMC methods only to simulate the posterior density of each parameter.

5.2 Simulation Results

We consider the same numerical example used by Nandi and Sreehari (1997) where $\pi = .10$, $p_1 = .002$, and $p_2 = .001$. As in case II, we generate 200 cycles in each simulation of the process.

The following two levels of prior information are considered:

- "Strong" (reasonably accurate information): $.001 \le p_1 \le .003$, $0 < p_2 \le .02$, $.07 \le \pi \le .13$
- "Weak" (moderately accurate information): $0 < p_1 \le .005$, $0 < p_2 \le .003$, $0 < \pi \le .25$

To study the sensitivity of the method with respect to the choice of the prior distributions, we consider the following three priors:

- Beta priors tightly distributed in the stated intervals with $\gamma_{p_1} = \gamma_{p_2} = \gamma_\pi = .05$, where γ is defined in the same way as in (9) and (11)
- Flatter Beta priors with $\gamma_{p_1} = \gamma_{p_2} = \gamma_\pi = .25$
- Uniform priors in the stated intervals.

We carried out 100 simulations for each of the 6 cases. The results, along with the hyperparameters of the prior distributions, are summarized in Table 7. Each estimate is the median of its simulated posterior distribution (10,000 MCMC iterations with burn-in of 1,000). Based on the results in Table 7, the following observations are:

1. The method works satisfactorily even when the prior information is not quite accurate.
2. As expected, the accuracy of prior information increases the efficiency of the parameter estimates. This is supported by the fact that the variances of the estimators are much lower with "strong" prior information than with "weak" information.
3. The method seems to be not very sensitive to the nature of prior distribution, as also seen in case II. If the prior information on π is of the form $\pi_0 \pm \delta$, then it might be easy to elicit a Beta prior with mean close to π_0. But, if this prior information is simply in the form of an interval π_L, π_U, then it would be more pragmatic to consider a uniform prior.

6. CONCLUDING REMARKS

In research into determining the optimum diagnosis interval for online monitoring of attributes, the issue of estimation of process parameters has not been given its just due. We have considered three different process models, designated case I, case II, and case III. Noting that the estimation problem is trivial for case I, we highlighted the problems associated with the estimation of the process parameters in case II (p and π) and case III (p_1, p_2, and π) and proposed a Bayesian method for this problem. We demonstrated the suitability of the proposed method in the two cases with extensive simulations and also through application to a real-world problem. The proposed method includes concrete guidelines for constructing prior distributions based on the available engineering knowledge.

Numerous authors have discussed the advantages and disadvantages of an economic approach to the design of control schemes. Woodall (1986, 1987) in particular has identified problems with the economic approach, criticizing it from two angles. First, most economic models of control charts have a high probability of type I error, thereby increasing the probability of false alarms. Second, economic models usually assign a cost to passing a defective characteristic, which includes liability claims and customer dissatisfaction costs, and it is counter to Deming's philosophy that these costs cannot be measured

Table 7. Case III: Summary of simulation results

Level of information	Prior distribution		π $\times 10^{-2}$	p_1 $\times 10^{-3}$	p_2 $\times 10^{-3}$
Strong	High-belief Beta	Mean	10.11	2.08	.88
	$\alpha_1 = 22, \alpha_2 = 4, \alpha_3 = 40,$	Median	10.10	2.07	.88
	$\beta_1 = 9980, \beta_2 = 4000, \beta_3 = 350$	Mode	9.99	2.06	.88
		Relative bias (%)	1.1	4.0	−12.0
		SD	.43	.22	.14
$.001 \le p_1 \le .003,$	Low-belief Beta	Mean	9.69	2.06	.93
$0 \le p_2 \le .002,$	$\alpha_1 = 5, \alpha_2 = .55, \alpha_3 = 12,$	Median	9.59	2.04	.94
$.07 \le \pi \le .13.$	$\beta_1 = 2500, \beta_2 = 400, \beta_3 = 110$	Mode	9.27	2.03	.94
		Relative bias (%)	−3.1	3.0	−7.0
		SD	1.14	.38	.24
	Uniform	Mean	9.82	2.00	.95
		Median	9.75	2.00	.97
		Mode	9.60	2.06	1.04
		Relative bias (%)	−1.8	0	−5.0
		SD	.74	.37	.22
Weak	High-belief Beta	Mean	10.00	2.17	.94
	$\alpha_1 = 22, \alpha_2 = 4, \alpha_3 = 40,$	Median	9.71	2.10	.94
	$\beta_1 = 9980, \beta_2 = 4000, \beta_3 = 450$	Mode	8.86	2.09	.95
		Relative bias (%)	0	8.5	−6.0
		SD	2.88	.55	.22
$0 \le p_1 \le .005,$	Low-belief Beta	Mean	10.05	2.43	.89
$0 \le p_2 \le .003,$	$\alpha_1 = 5, \alpha_2 = .55, \alpha_3 = 12,$	Median	8.87	2.26	.91
$0 \le \pi \le .25.$	$\beta_1 = 2500, \beta_2 = 400, \beta_3 = 110$	Mode	6.88	2.15	.99
		Relative bias (%)	.5	21.5	−11.0
		SD	4.83	.77	.31
	Uniform	Mean	9.63	2.45	.94
		Median	8.68	2.27	.95
		Mode	7.32	2.15	1.00
		Relative bias (%)	−3.7	22.5	−6.0
		SD	4.32	.75	.29

and that customer satisfaction is necessary to staying in business.

The cost component C_D may be difficult to measure, particularly when it relates to an external customer. This cost is an external failure cost and may include warranty claims, warranty repairs/replacements, product recalls, and product liability (Ross 1993), which usually are available from accounting records. Measuring external failure cost associated with lost sales or customer dissatisfaction is not straightforward, however. A few authors have addressed the issue of estimating these costs. One approach is to use Taguchi's loss function (Ross 1993; Margavio, Fink, and Margavio 1994). Giakatis, Enkawa, and Washitani (2001) classified external failure cost as quality loss, further establishing this link. Taguchi's (1986) quadratic loss function is a nice way of quantifying the financial loss a customer incurs when a nonacceptable product is passed on. Several authors have proposed alternatives to Taguchi's loss functions; for example, Joseph (2004) derived a set of loss functions for nonnegative variables and discussed how these functions can be estimated. Other approaches of quantifying customer dissatisfaction cost include projection (projecting the financial impact of customer problem experiences) and method of direct linkage (Vavra 1997). But such approaches may not be capable of assessing the financial impact of a single defective item in terms of customer dissatisfaction, but may help in estimating hidden components of customer dissatisfaction.

Recalling the two assumptions stated at the beginning of Section 2, it is clear that we have considered situations in which neither type I nor type II errors are likely to occur. Therefore, the control problem discussed here is different from the classical control charting problem.

The biggest advantage of economic model is perhaps its ability to convince top management by projecting the benefits of process improvement in terms of hard cash. As emphasized by Taguchi et al. (1989) and Nayebpour and Woodall (1993), such a study should pave the way for continuous improvement.

An interesting topic of future research in this area is to develop a generic framework with k types of assignable causes that would have cases I, II, and III as special cases. This is encountered in several industrial situations.

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APPENDIX A: TECHNICAL RESULTS

Estimation of p and π Following Srivastava and Wu (1993)

Let us assume that $l = 0$ and $s_i \geq 2$ for $i = 1, 2, \dots, N$. Note that K denotes the total number of inspections, which, according to our notation, is $\sum_{i=1}^{N} s_i$. Because $n = 1$, we have that

$$\psi_k = \begin{cases} p & \text{if } x_{k-1} = 0 \\ p\pi & \text{if } x_{k-1} = 1. \end{cases}$$

From (6), we get

$$\begin{aligned} 1 &= \frac{1}{\sum_{i=1}^{N} s_i} \left(\sum_{\{k:x_k=1\}} \frac{1}{\psi_k} \right) \\ &= \frac{1}{\sum_{i=1}^{N} s_i} \left(\sum_{\{k:x_k=1,x_{k-1}=0\}} \frac{1}{\psi_k} \right) \\ &\quad \text{[because by the assumption } \min(s_i) \geq 2, \\ &\quad \text{it follows that } x_k = 1 \Leftrightarrow x_{k-1} = 0] \\ &= \frac{1}{\sum_{i=1}^{N} s_i} \left(\sum_{\{k:x_k=1,x_{k-1}=0\}} \frac{1}{p} \right). \end{aligned}$$

Noting that cardinality of the set $\{k : x_k = 1\}$ is N, we get

$$\hat{p} = \frac{N}{\sum_{i=1}^{N} s_i}. \tag{A.1}$$

Again, from (7), it follows that

$$\begin{aligned} 1 &= \frac{1}{\sum_{i=1}^{N} s_i} \left(\sum_{\{k:x_k=0\}} \frac{1}{1-\psi_k} \right) \\ &= \frac{1}{\sum_{i=1}^{N} s_i} \\ &\quad \times \left[\sum_{\{k:x_k=0,x_{k-1}=0\}} \frac{1}{1-\psi_k} + \sum_{\{k:x_k=0,x_{k-1}=1\}} \frac{1}{1-\psi_k} \right] \\ &= \frac{1}{\sum_{i=1}^{N} s_i} \\ &\quad \times \left[\sum_{\{k:x_k=0,x_{k-1}=0\}} \frac{1}{1-p} + \sum_{\{k:x_k=0,x_{k-1}=1\}} \frac{1}{1-p\pi} \right]. \end{aligned}$$

Again, by the assumption $\min(s_i) \geq 2$, the cardinalities of the sets $\{k : x_k = 0, x_{k-1} = 1\}$ and $\{k : x_k = 0, x_{k-1} = 0\}$ are $(N - 1)$ and $(\sum_{i=1}^{N} s_i - 2N + 1)$. Thus

$$\frac{\sum_{i=1}^{N} s_i - 2N + 1}{1-p} + \frac{N-1}{1-p\pi} = \sum_{i=1}^{N} s_i, \tag{A.2}$$

and, substituting $\hat{p} = \frac{N}{\sum_{i=1}^{N} s_i}$, we get $\hat{\pi} = 1$.

Proof of Proposition 1

For $s \geq 1$, we have

$$\begin{aligned} P(S = s|p,\pi) &= P(X + Y + [l/m] = s|p,\pi) \\ &= \sum_{k=1}^{r} P\big(X = k \cap Y = r-k\big), \\ &\quad \text{where } r = s - [l/m] \\ &= \sum_{k=1}^{r} P(X = k)P(Y = r-k) \\ &= \sum_{k=1}^{r} q^{km-m}(1-q^m)\pi(1-\pi)^{r-k} \\ &= (1-q^m)\pi(1-\pi)^{r-1} \sum_{k=1}^{r} \frac{q^{(k-1)m}}{(1-\pi)^{k-1}} \\ &= \pi(1-q^m)\frac{(1-\pi)^r - q^{rm}}{1-\pi-q^m}. \end{aligned}$$

Because for $r > 0$, $(1-\pi)^r - q^{rm} \gtreqless 0$ according as $(1-\pi) - q^m \gtreqless 0$, the result follows immediately.

Proof of Proposition 2

Before deriving the likelihood of the data, we introduce some additional notation.

- E, the event that the first defect was one of the minor type [Note that $P(E) = (p_1 - p_1p_2)/(p_1 + p_2 - p_1p_2) = \alpha$, say.]
- $\xi_{y,1}$, the probability that a minor defect is detected at the yth sampled observation after the defect had occurred, given that the first defect was one of the minor type
- $\xi_{y,2}$, the probability that a major defect is detected at the yth sampled observation after the defect had occurred, given that the first defect was one of the minor type
- ξ_y, the probability that the defect is discovered at the yth sampled observation after the defect had occurred, given that the first defect was one of the minor type [Note that $\xi_y = P(Y = y-1|E)$ and $\xi_y = \xi_{y,1} + \xi_{y,2}$.]
- B_y, the event that the major defect occurs in the yth diagnosis interval given that the first defect was of the minor type.

Now,

$$\begin{aligned} P(S = s) &= \sum_{k=1}^{r} P(X = k)P(Y = r-k), \quad \text{where } r = s - [l/m] \\ &= \sum_{k=1}^{r} P(X = k)\big(P(Y = r-k|E)P(E) \\ &\quad + P(Y = r-k|\overline{E})P(\overline{E})\big). \end{aligned} \tag{A.3}$$

Noting that

$$P(Y=y|\overline{E}) = \begin{cases} 1 & \text{for } y=0 \\ 0 & \text{for } y>0, \end{cases}$$

from (A.3), we can write

$$\begin{aligned} P(S=s) &= \sum_{k=1}^{r-1} P(X=k)P(Y=r-k|E)P(E) \\ &\quad + P(X=r)\big(P(Y=0|E)P(E)+P(\overline{E})\big) \\ &= \sum_{k=1}^{r-1} P(X=k)\xi_{r-k+1}P(E) \\ &\quad + P(X=r)(\xi_1 P(E)+P(\overline{E})) \\ &= \alpha\sum_{k=1}^{r-1} P(X=k)\xi_{r-k+1} \\ &\quad + P(X=r)(\alpha\xi_1+1-\alpha). \end{aligned} \quad \text{(A.4)}$$

Using the results of Nandi and Sreehari (1997), we obtain

$$P(X=x) = q^{xm-m}(1-q^m), \quad \text{where } q=q_1q_2; \quad \text{(A.5)}$$

$$\begin{aligned} P(B_1) &= \frac{1}{1-q_1^m}\sum_{k=0}^{m-1} q_1^k p_1(1-q_2^{m-k}) \\ &= 1-\frac{p_1q_2(q_2^m-q_1^m)}{(1-q_1^m)(q_2-q_1)} = 1-\Delta; \end{aligned} \quad \text{(A.6)}$$

$$\begin{aligned} P(B_y) &= \frac{1}{1-q_1^m}\sum_{k=0}^{m-1} q_1^k p_1 q_2^{(y-1)m-k}(1-q_2^m) \\ &= \frac{p_1q_2(1-q_2^m)(q_2^m-q_1^m)}{(1-q_1^m)(q_2-q_1)}q_2^{(y-2)m} \quad \text{for } y=2,3,\ldots; \\ &= \Delta(1-q_2^m)q_2^{(y-2)m} \quad \text{for } y=2,3,\ldots; \end{aligned} \quad \text{(A.7)}$$

$$\xi_{y,1} = \big(1-P(B_1)-P(B_2)-\cdots-P(B_y)\big)(1-\pi)^{y-1}\pi, \quad y=1,2,\ldots; \quad \text{(A.8)}$$

and

$$\xi_{y,2} = (1-\pi)^{y-1}P(B_y), \qquad y=1,2,\ldots. \quad \text{(A.9)}$$

Therefore,

$$\xi_{y,1} = \begin{cases} \Delta\pi & \text{for } y=1 \\ \Delta\left(1-\dfrac{1-q_2^m}{p_2}\big(1-q_2^{(y-1)m}\big)\right)(1-\pi)^{y-1}\pi & \\ \qquad \text{for } y>1, & \end{cases}$$

$$\xi_{y,2} = \begin{cases} 1-\Delta & \text{for } y=1 \\ \Delta(1-q_2^m)q_2^{(y-2)m}(1-\pi)^{y-1} & \text{for } y>1, \end{cases}$$

and, consequently,

$$\xi_y = \begin{cases} 1-\Delta(1-\pi) & \text{for } y=1 \\ \Delta(1-\pi)^{y-1}\left[\pi\left(1-\dfrac{1-q_2^m}{p_2}\big(1-q_2^{(y-1)m}\big)\right)\right. & \\ \qquad \left. + (1-q_2^m)q_2^{(y-2)m}\right] & \text{for } y>1. \end{cases}$$

Plugging in the foregoing expression for ξ_y and (A.5) into (A.4), the result follows after some tedious algebraic manipulations.

APPENDIX B: EFFECT OF INSPECTION ERROR ON THE ESTIMATION

Table A.1. Summary of simulation output for π: Effect of inspection error

				b							
				.005		.010		.050		.100	
a	Available information	Method		Mean $\times 10^{-2}$	SD $\times 10^{-2}$	Mean $\times 10^{-2}$	SD $\times 10^{-2}$	Mean $\times 10^{-2}$	SD $\times 10^{-2}$	Mean $\times 10^{-2}$	SD $\times 10^{-2}$
.001	$0 \le \pi \le .15$ No information on p	Uniform		10.33	1.04	10.37	1.13	9.82	1.09	9.15	.96
		Beta	Tight	10.04	.67	10.09	.74	9.69	.73	9.19	.68
			Flat	10.72	1.04	10.76	1.15	10.65	1.11	10.11	1.12
	$.07 \le \pi \le .17$ $0 < p \le .0005$	Uniform		10.34	.94	10.41	1.04	9.89	.86	9.62	.96
		Beta	Tight	10.41	.70	10.44	.74	10.04	.63	9.80	.76
			Flat	10.94	.99	10.78	.99	10.24	1.02	9.84	1.00
.005	$0 \le \pi \le .15$ No information on p	Uniform		10.41	1.07	10.35	.97	10.08	1.12	9.42	.96
		Beta	Tight	10.20	.66	10.15	.67	9.92	.75	9.44	.67
			Flat	10.97	1.06	10.76	1.15	10.66	1.15	10.21	1.21
	$.07 \le \pi \le .17$ $0 < p \le .0005$	Uniform		10.86	1.07	10.93	1.09	10.18	1.01	9.84	1.02
		Beta	Tight	10.86	.77	10.91	.77	10.31	.74	10.01	.79
			Flat	10.81	.94	10.84	1.00	10.49	.99	10.13	1.06

Table A.1. (Continued)

				b = .005		.010		.050		.100	
a	Available information	Method		Mean $\times 10^{-2}$	SD $\times 10^{-2}$	Mean $\times 10^{-2}$	SD $\times 10^{-2}$	Mean $\times 10^{-2}$	SD $\times 10^{-2}$	Mean $\times 10^{-2}$	SD $\times 10^{-2}$
.010	$0 \le \pi \le .15$ No information on p	Uniform		10.60	1.04	10.59	1.06	10.33	1.12	10.12	1.05
		Beta	Tight	10.45	.67	10.45	.69	10.21	.75	10.01	.73
			Flat	11.28	.98	11.23	1.06	10.75	1.07	10.53	1.11
	$.07 \le \pi \le .17$ $0 < p \le .0005$	Uniform		11.40	1.10	11.46	1.12	10.85	.97	10.29	.99
		Beta	Tight	11.32	.81	11.35	.77	10.90	.74	10.41	.76
			Flat	11.41	1.09	11.39	1.03	10.73	.99	10.42	1.06
.050	$0 \le \pi \le .15$ No information on p	Uniform		13.47	.60	13.42	.57	13.10	.76	12.80	.77
		Beta	Tight	13.39	.77	13.33	.75	13.01	.88	12.67	.75
			Flat	13.45	1.09	13.51	.97	13.22	1.07	13.03	.93
	$.07 \le \pi \le .17$ $0 < p \le .0005$	Uniform		15.40	.68	15.37	.77	15.11	.79	14.69	.96
		Beta	Tight	15.06	.97	14.99	.94	14.77	.95	14.27	1.09
			Flat	14.55	1.08	14.65	1.18	14.14	1.01	13.77	1.16

Table A.2. Summary of simulation output for p: Effect of inspection error

				b = .005		.010		.050		.100	
a	Available information	Method		Mean $\times 10^{-4}$	SD $\times 10^{-5}$	Mean $\times 10^{-4}$	SD $\times 10^{-5}$	Mean $\times 10^{-4}$	SD $\times 10^{-5}$	Mean $\times 10^{-4}$	SD $\times 10^{-5}$
.001	$0 \le \pi \le .15$ No information on p	Uniform		3.54	9.26	3.59	10.28	3.67	10.81	3.82	10.46
		Beta	Tight	3.79	6.72	3.78	6.98	3.80	7.64	3.83	7.43
			Flat	3.36	10.31	3.41	9.97	3.11	8.28	3.19	9.87
	$.07 \le \pi \le .17$ $0 < p \le .0005$	Uniform		3.40	5.44	3.36	5.63	3.44	5.62	3.39	5.92
		Beta	Tight	3.33	3.44	3.32	3.64	3.33	3.71	3.27	3.86
			Flat	3.07	6.19	3.15	6.10	3.19	6.57	3.21	6.90
.005	$0 \le \pi \le .15$ No information on p	Uniform		4.10	11.82	3.88	9.67	3.98	11.43	4.11	11.46
		Beta	Tight	4.20	7.78	4.06	6.41	4.08	7.56	4.08	7.89
			Flat	3.57	10.10	3.74	10.40	3.54	11.20	3.59	10.46
	$.07 \le \pi \le .17$ $0 < p \le .0005$	Uniform		3.44	5.73	3.43	5.47	3.58	5.49	3.56	5.66
		Beta	Tight	3.39	3.51	3.39	3.44	3.45	3.65	3.41	3.72
			Flat	3.56	6.12	3.55	6.97	3.47	6.93	3.36	8.02
.010	$0 \le \pi \le .15$ No information on p	Uniform		4.56	11.98	4.56	12.90	4.42	12.09	4.22	11.37
		Beta	Tight	4.57	7.82	4.54	8.65	4.42	7.50	4.30	7.66
			Flat	3.87	9.68	3.93	11.52	4.08	11.00	3.86	11.96
	$.07 \le \pi \le .17$ $0 < p \le .0005$	Uniform		3.62	5.95	3.57	5.82	3.67	5.43	3.63	5.51
		Beta	Tight	3.60	4.29	3.53	3.68	3.57	3.73	3.50	3.82
			Flat	3.61	7.75	3.59	7.22	3.73	6.68	3.69	7.99
.050	$0 \le \pi \le .15$ No information on p	Uniform		7.25	8.20	7.22	7.63	7.21	7.13	7.06	8.09
		Beta	Tight	7.00	10.85	6.95	9.95	6.92	8.96	6.80	9.63
			Flat	7.52	18.76	7.60	20.19	7.25	17.72	7.34	18.39
	$.07 \le \pi \le .17$ $0 < p \le .0005$	Uniform		4.55	1.92	4.56	1.84	4.51	1.75	4.51	2.04
		Beta	Tight	4.67	4.39	4.68	4.14	4.55	3.51	4.63	3.86
			Flat	5.53	9.64	5.35	8.44	5.58	8.62	5.56	9.62

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REFERENCES

Adams, B. M., and Woodall, W. H. (1989), "An Analysis of Taguchi's On-Line Process Control Procedure Under a Random Walk Model," *Technometrics*, 31, 401–413.

Barlow, R. E., and Proschan, F. (1965), *Mathematical Theory of Reliability*, New York: Wiley.

Borges, W., Ho, L. L., and Turnes, O. (2001), "An Analysis of Taguchi's On-Line Quality Monitoring Procedure for Attributes With Diagnosis Errors," *Applied Stochastic Models in Business and Industry*, 17, 261–276.

Brown, M., and Proschan, F. (1983),"Imperfect Repair," *Journal of Applied Probability*, 20, 851–859.

Dasgupta, T. (2003), "An Economic Inspection Interval for Control of Defective Items in a Hot Rolling Mill," *Journal of Applied Statistics*, 30, 273–282.

Elsayed, E. A., and Okumoto, K. (1983), "An Optimum Group Maintenance Policy," *Naval Research Logistics Quarterly*, 30, 667–674.

Esary, J. D., Marshall, A. W., and Proschan, F. (1973), "Shock Models and Wear Processes," *The Annals of Probability*, 1, 627–649.

Fink, R. L. (1993), "Quality Improvement Technology Using the Taguchi Method," *CPA Journal Online*, December 1993, available at *http://www.nysscpa.org/cpajournal/old/14903851.htm*.

Gelman, A., Carlin, J. B., Stern, H. S., and Rubin, D. B. (2004), *Bayesian Data Analysis*, Boca Raton: FL: CRC Press.

Giakatis, G., Enkawa, T., and Washitani, K. (2001), "Hidden Quality Costs and the Distinction Between Quality Cost and Quality Loss," *Total Quality Management*, 12, 179–190.

Joseph, V. R. (2004), "Quality Loss Functions for Nonnegative Variables and Their Applications," *Journal of Quality Technology*, 36, 129–138.

Margavio, G. W., Fink, R. L., and Margavio, T. M. (1994), "Quality Improvement Using Capital Budgeting and Taguchi's Function," *International Journal of Quality and Reliability Management*, 11, 10–20.

Nakagawa, T. (1988), "Sequential Imperfect Preventive Maintenance Policy," *IEEE Transactions on Reliability*, 37, 295–298.

Nandi, S. N., and Sreehari, M. (1997), "Economy Based On-Line Quality Control Method for Attributes," *Sankhyā: The Indian Journal of Statistics*, Ser. B, 59, 384–395.

Nayebpour, M. R., and Woodall, W. H. (1993), "An Analysis of Taguchi's On-Line Quality Monitoring Procedures for Attributes," *Technometrics*, 35, 53–60.

R Development Team (2006), *R: A Project for Statistical Computing*, version 2.0, available at *http://www.r-project.org/*.

Ross, F. L. (1993), "Quality Improvement Technology Using the Taguchi Method (CPA in Industry)," *The CPA Journal Online*, December, available at *http://www.nysscpa.org/cpajournal/old/14903851.htm*.

Ross, S. M. (1996), *Stochastic Processes*, New York: Wiley.

Srivastava, M. S., and Wu, Y. (1991), "A Second Order Approximation to Taguchi's On-Line Control Procedure," *Communications in Statistics, Part A—Theories and Methods*, 20, 2149–2168.

——— (1993), "Estimation and Testing in an Imperfect-Inspection Model," *IEEE Transactions on Reliability*, 42, 280–286.

——— (1994), "On-Line Control Procedures Under the Random Walk Model With Measurement Error and Attribute Observations," *Canadian Journal of Statistics*, 22, 377–386.

——— (1995), "An Improved Version of Taguchi's On-Line Control Procedure," *Journal of Statistical Planning and Inference*, 43, 133–145.

——— (1996), "Economical Quality Control Procedures Based on Symmetric Random Walk Model," *Statistica Sinica*, 6, 389–402.

Taguchi, G. (1981), *On-Line Quality Control During Production*, Tokyo: Japanese Standards Association.

——— (1984), *Quality Evaluation for Quality Assurance*, Dearborn, MI: American Supplier Institute.

——— (1985), "Quality Engineering in Japan," *Communications in Statistics, Part A—Theory and Methods*, 14, 2785–2801.

——— (1986), *Introduction to Quality Engineering: Designing Quality Into Products and Processes*, White Plains, NY: Asian Productivity Organization, UNIPUB/Kraus International.

Taguchi, G., Elsayed, E. A., and Hsiang, T. (1989), *Quality Engineering in Production Systems*, New York: McGraw-Hill.

Vavra, T. G. (1997), *Improving Your Measurement of Customer Satisfaction*, Milwaukee, WI: ASQC Quality Press.

Woodall, W. H. (1986), "Weaknesses of Economic Design of Control Charts" [letter to the editor], *Technometrics*, 28, 408–410.

——— (1987), "Conflicts Between Deming's Philosophy and Economic Design of Control Charts," in *Frontiers of Statistical Quality Control 3*, eds. H. J. Lenz, G. B. Wetherill, and P. T. Wilrich, Heidelberg: Physica-Verlag, pp. 242–248.